Comment on "Low-frequency character of the Casimir force between metallic films"

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(Received 2 December 2005; published 10 April 2006)

In Phys. Rev. E **70**, 047102 (2004), Torgerson and Lamoreaux investigated for the first time the real-frequency spectrum of the finite temperature correction to the Casimir force, for metallic plates of finite conductivity. The very interesting result of this study is that the large correction from the TE mode is dominated by low frequencies, for which the dielectric description of the metal is invalid, and the authors correctly point out that a more realistic description is provided by low-frequency metallic boundary conditions. However, their subsequent analysis uses an incorrect form of metallic boundary conditions for TE modes. After correcting this error, we find that their main conclusion was nevertheless qualitatively correct: contrary to the result of the dielectric model, the thermal TE mode correction leads to an increase in the TE mode force of attraction between the plates. The correction found by us, however, has a magnitude about 20 times larger than that quoted by Torgerson and Lamoreaux.

DOI: 10.1103/PhysRevE.73.048101 PACS number(s): 05.30.−d, 12.20.Ds, 41.20.−q, 12.20.Fv

In the recent literature on the Casimir effect, much attention has been devoted to the issue of evaluating the corrections to the Casimir force between metallic bodies, arising from the combined effect of temperature and finite conductivity of the plates. An estimate of these corrections [1], using a dielectric Drude model (with dissipation) for the plates, leads to surprisingly large deviations from the perfectly conducting case, for separations among the plates greater than a micrometer or so, at room temperature. Several authors have criticized the validity of these results, for different reasons. Torgerson and Lamoreaux [2], in particular, performed for the first time a spectral analysis of these thermal corrections along the *real-frequency axis*, while standard treatments based on Lifshitz theory always deal with imaginary frequencies, which have a far less clear physical meaning. The new result of this very interesting study is that the large corrections found in [1] arise from TE *evanescent modes of low frequencies*. The frequencies involved are sufficiently low for the dielectric description of the metal to be invalid. Torgerson and Lamoreaux correctly suggest that a more realistic description of the metal, in the frequency region of interest, can be obtained in terms of Leontovich surface impedance boundary conditions (BC). Following the notations of [2], we assume that the plates' surfaces are at $z=0$ and $z=a$. Then, for a TE mode of frequency ω, propagating along the x axis, the BC for a good conductor read as

$$E_y = \pm \zeta H_x, \tag{1}$$

where the + and − refer to $z=0$ and to $z=a$, respectively. For the surface impedance ζ, Torgerson and Lamoreaux use the following expression,

$$\zeta = (1-i)\sqrt{\frac{\omega}{8\pi\sigma}}, \tag{2}$$

which is valid for frequencies in the normal skin-effect region. However, at this point, Torgerson and Lamoreaux use an incorrect form of the fourth Maxwell's equation in the vacuum, their Eq. (8), which does not take account of the z component of the magnetic field. Indeed, the magnetic field present in the empty gap between the plates can be obtained from the second Maxwell equation,

$$\vec{\nabla} \times \vec{E} - i\frac{\omega}{c}\vec{B} = \vec{0}. \tag{3}$$

For TE modes, with $\vec{E}=\hat{y}E_y$, one obtains

$$\vec{B} = \frac{c}{\omega}\left(\hat{z}kE_y + i\hat{x}\frac{\partial E_y}{\partial z}\right), \tag{4}$$

where $\vec{k}$ is the transverse wave vector. We see that the magnetic field has a z component $B_z=ckE_y/\omega$, which was omitted in Eq. (8) of [2], whose correct form really is (for $\mu=1$),

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -\frac{i\omega}{c}E_y. \tag{5}$$

As a consequence of this error, the BC on the magnetic field given in Eq. (9) of [2] are incorrect. In fact the correct BC are best written in terms of the electric field. By using the expression of H_x in terms of E_y, obtained from Eq. (4) above, we can rewrite the impedance BC for TE modes in Eq. (1) as

$$E_y = \pm \frac{ic\zeta}{\omega}\frac{\partial E_y}{\partial z}. \tag{6}$$

If one defines the spectrum F_ω of the thermal correction to the Casimir force F by the equation,

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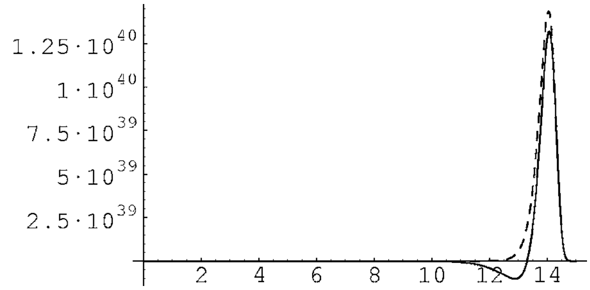

FIG. 1. Plots of the contribution to F_ω from the C_1 path (plane waves), for perfectly conducting plates (dashed line) and for finite conductivity boundary conditions, as functions of $\log_{10}(\omega)$. All are for $a=1$ μm, $T=300$ K. Treatment of the plates as conducting metals fails above $\omega=10^{14}$ rad/s.

$$F = \frac{\hbar}{\pi^2 c^3}\int_0^\infty d\omega\, F_\omega \tag{7}$$

(attraction corresponds to $F>0$) by simple computations analogous to those after Eq. (9) of [2], one can get the following expression for the TE-mode contribution to $F_\omega^{(TE)}$, in the simple case of two identical plates,

$$F_\omega^{(TE)} = \omega^3 g(\omega)\mathrm{Re}\int_C p^2\, dp\left[\left(\frac{1+\zeta p}{1-\zeta p}\right)^2 e^{-2i\omega pa/c} - 1\right]^{-1}, \tag{8}$$

which should be used in the place of Eq. (11) of [2]. We note that Eq. (11) of [2], in fact, accidentally reproduces, the TM modes contribution to F_ω,

$$F_\omega^{(TM)} = \omega^3 g(\omega)\mathrm{Re}\int_C p^2\, dp\left[\left(\frac{p+\zeta}{p-\zeta}\right)^2 e^{-2i\omega pa/c} - 1\right]^{-1}. \tag{9}$$

Following Torgerson and Lamoreaux, the integration path is separated into C_1 for $p=1$ to 0 (which describes the contribution from the plane waves), and C_2 with p pure imaginary from $p=i0$ to $i\infty$ (corresponding to evanescent waves). In our computations, we used for σ the expression,

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\tau\omega}, \tag{10}$$

with $\sigma_0=3\times 10^{17}$ s^{-1} and $\tau=1.88\times 10^{-14}$ s, which are the values for Au. We found that, both in the transverse magnetic sector and in the plane-wave TE sector, the spectra obtained from Eqs. (8) and (9) coincide, to a high degree of accuracy, with those derived from dielectric BC. For $T=300$ K and $a=1$ μm, the two approaches lead in these sectors to integrated forces that differ by a few parts in a thousand. Significant differences are found only in the evanescent TE sector. Results of the numerical integration of Eq. (8), for $a=1$ μm, $T=300$ K are shown in Figs. 1 and 2. We see from Fig. 2 that the thermal correction from evanescent modes has a *positive* sign, which means that it represents an *attractive* contribution, contrary to the result obtained from dielectric BC (see Fig. 1 of [2]), and in agreement with what was reported by Torgerson and Lamoreaux. The integrated force for the C_2 path is 38 times larger than the C_1 integration, while Torgerson and Lamoreaux reported a result only 1.47 times greater. The total net force for both paths is 36.5 times larger than the perfectly conducting case, while the above authors obtained a result 1.75 larger. As discussed in [2] treatment of the plates as good conductors is not valid above $\omega=10^{14}$ rad/s.

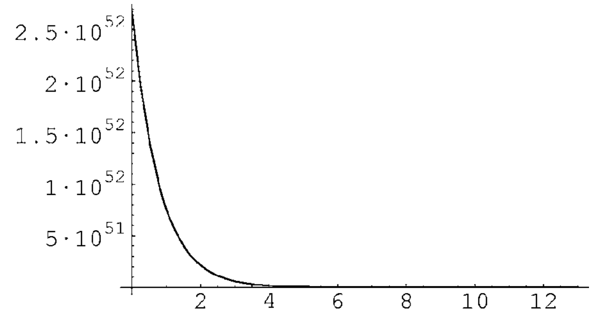

FIG. 2. Plot of the contribution to F_ω from the C_2 path (evanescent waves), for finite conductivity boundary conditions, as a function of $\log_{10}(\omega)$, for $a=1$ μm, $T=300$ K. The integrated force is *attractive* and has a magnitude 38 times larger than the C_1 integration. The total net force for both paths is 36.5 times greater than the perfectly conducting case. Treatment of the plates as conducting metals fails above $\omega=10^{14}$ rad/s.

Our conclusion is that, despite the error in the BC, the qualitative results of Ref. [2] are correct: if one models the plates as good conductors, one finds that the TE mode thermal correction leads to an increase in the TE mode force, contrary to what is obtained from the dielectric model, and the magnitude of the correction is over 35 times larger than the perfectly conducting case. Finally, we remark that Ref. [3] reports the same erroneous form of impedance BC for the TE modes as that of [2].

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